FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
OPEN TEST – II
PAPER –2
TEST DATE: 21-04-2024

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 51 questions.
- Each subject (PCM) has 17 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section-B & Section-C**.
  Section – A (01 – 04, 18 – 21, 35 – 38): *This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*
  Section – A (05 –07, 22 – 24, 39 – 41): *This section contains **NINE (09)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).*
  Section – B (08 – 13, 25 – 30, 42 – 47): *This section contains **EIGHTEEN (18)** numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*
  Section – C (14 –17, 31 – 34, 48 – 51): *This section contains **SIX (06) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

MARKING SCHEME

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	Question is unanswered;
Negative Marks	:	0	In all other cases.

Section – C: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct integer is entered;
Zero Marks	:	0	Question is unanswered;
Negative Marks	:	0	In all other cases.

Physics

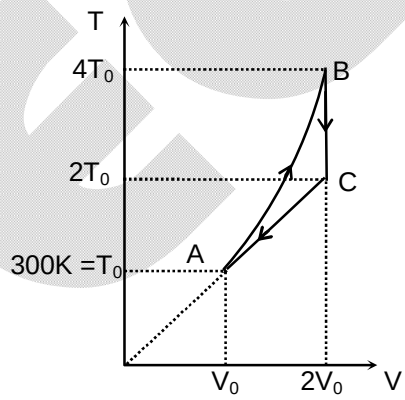
PART – I

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

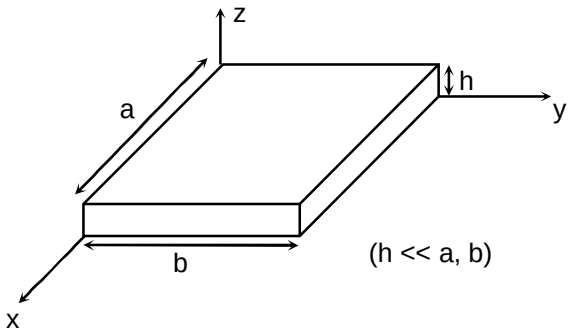
1. A thin uniform square plate of mass m = 1 kg and side a = 1m can freely rotate about a stationary vertical axis coinciding with one of its sides. A small ball of mass m = 1 kg moving horizontally with a velocity v = 14 m/s at right angle to the plane of the plate collides elastically at the centre of the plate. The angular velocity of the plate just after collision is
(A) 12 rad/s (B) 24 rad/s
(C) 36 rad/s (D) 48 rad/s

2. One mole of an ideal monoatomic gas is taken through a cyclic process ABCA as shown in the T-V diagram. In the process AB, $T = \alpha V^2$, where α is a positive constant. The efficiency of the given cycle in percentage is
(A) 8.33
(B) 16.33
(C) 24.33
(D) 28.33



3. One half of a rod of length ℓ = 100 cm has density ρ_1 = 0.50 g/cm^3 and the other half has density ρ_2 = 2.50 g/cm^3. The rod is placed horizontally on the horizontal bottom of a large tub and then water is gradually poured in the tub. Find the depth of water in the tub, when the rod makes an angle $\theta = 53°$ with the horizontal The density of water is ρ = 1 g/cm^3.
(A) 20 cm (B) 40 cm
(C) 60 cm (D) 80 cm

4. The top and bottom faces of a rigid insulating box of dimensions a × b × h and mass m are charged with uniform surface charge densities $+\sigma$ and $-\sigma$ respectively and the box is placed in a uniform horizontal magnetic field $\vec{B} = B_0\hat{j}$. Assuming free space conditions and h is negligibly small as compared to the dimensions a and b. After having the magnetic field switched off, the velocity acquired by the insulating box will be

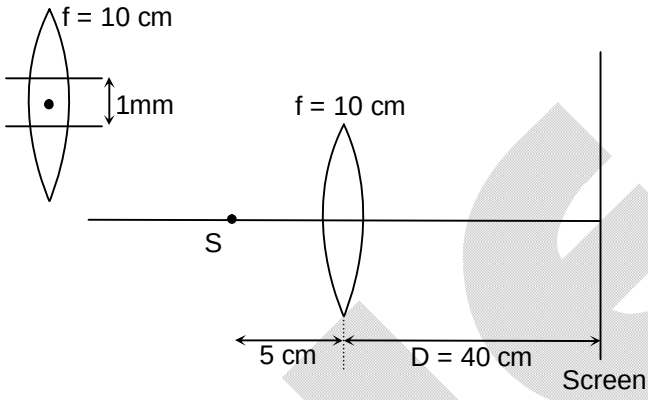


(A) $\left(\frac{\sigma abhB_0}{m}\right)\hat{j}$ (B) $\left(\frac{2\sigma abhB_0}{m}\right)\hat{j}$
(C) $\left(\frac{\sigma abhB_0}{m}\right)\hat{i}$ (D) $\left(\frac{2\sigma abhB_0}{m}\right)\hat{i}$

SECTION – A
(One or More than one correct type)

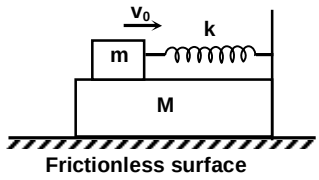
*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

5. A central portion of width d = 1 mm is cut out of a converging lens of focal length f = 10 cm symmetrically and the two halves are tightly fitted against each other and placed at a distance D = 40 cm from a screen as shown in the figure. A monochromatic point source S emitting light of wavelength λ = 4000 Å is placed at a distance 5 cm from the lens. Then choose the correct option(s).



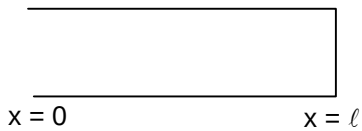
(A) The number of interference fringes obtained on the screen will be n = 20.
(B) The number of interference fringes obtained on the screen will be n = 40.
(C) The fringe width obtained on the screen is ω = 0.20 mm.
(D) The fringe width obtained on the screen is ω = 0.40 mm.

6. A block of mass m = 1 kg is placed on a plank of mass M = 3 kg which is kept on a smooth horizontal surface. The block is connected to the right end of the plank with the help of a light spring of stiffness k = 100 N/m as shown in the figure. The coefficient of friction between the block and the plank is μ = 0.5. Initially when the spring is relaxed, the block is given a velocity v_0 towards the right end of the plank. If the block stops on the plank before reversing its direction of motion relative to the plank. Then choose the correct option(s). (Take g = 10 m/s^2)



(A) The initial velocity of the block is v_0 = 2 m/s
(B) The initial velocity of the block is v_0 = 1 m/s
(C) The total work done by the friction on the system is (–0.25 J)
(D) The total work done by the friction on the system is (–0.75 J)

7. The air column in a pipe closed at one end is made to vibrate in its third overtone by a tuning fork of frequency 660 Hz. The velocity of sound in air is 330 m/s. If P_0 represents the mean pressure and ΔP_0 the maximum amplitude of pressure variation in the pipe. Neglect the end correction. Then choose the correct option(s).



(A) The length of the pipe is $\ell = \frac{7}{8}$m

(B) The maximum pressure at x = $\frac{\ell}{3}$ will be $\left(P_0 + \frac{\Delta P_0}{2}\right)$

(C) The maximum pressure at x = $\frac{\ell}{3}$ will be $\left(P_0 + \frac{\sqrt{3}\Delta P_0}{2}\right)$

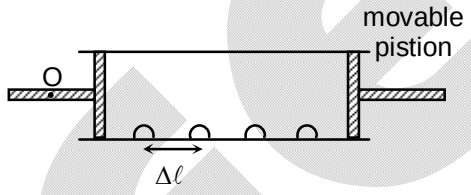
(D) The minimum pressure at x = $\frac{\ell}{2}$ will be $\left(P_0 - \frac{\Delta P_0}{\sqrt{2}}\right)$

SECTION – B

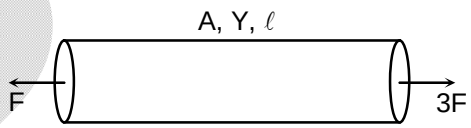
(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

8. In Kundt's tube method, a cylinder is closed at one end by a thin disc with a rod of length ℓ = 1.2 m is rigidly attached to it and other end is closed by a movable piston. Helium gas is enclosed in the cylinder and powder is sprinkled inside it. The rod is rigidly clamped at its mid-point and longitudinal standing waves are produced in the rod, so that the rod is vibrating with its first overtone frequency. When the gas column in the cylinder resonates with the rod, the separation between the two successive heaps of the powder formed is $\Delta\ell$ = 10 cm. Find the temperature (in Kelvin) of the gas in the cylinder. ($R = \frac{25}{3}$ Joule/mol-K, $Y_{rod} = 2\times10^{11}$ N/m^2 and $\rho_{rod} = 8000$ kg/m^3)



9. A uniform rod of length ℓ and Young's modulus Y is subjected to two opposite horizontal forces F and 3F at the two ends as shown in the figure. If the ratio of elastic potential energy stored in the right half to that in the left half of the rod is $\left(\frac{K}{7}\right)$. Find the value of K.

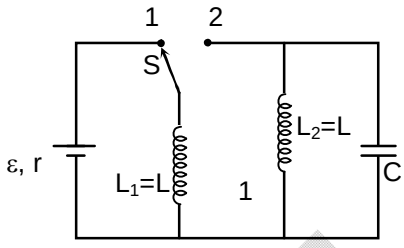


10. A heat conducting piston connected with the help of a spring to one end of a horizontal closed insulated cylinder, divides the cylinder into two parts each of which contains equal number of moles of an ideal monoatomic gas at an initial temperature T_0 = 675 K. The length of the cylinder is 2ℓ and relaxed length of the spring is $\frac{\ell}{2}$. Initially, elongation in the spring is $\frac{\ell}{4}$ and the piston is in equilibrium. Now, a small hole is made in the piston. If heat capacities of the cylinder, the piston and the spring are negligible then find the change in the temperature (in Kelvin) of the gas after new equilibrium is established.

11. In a setup of displacement method experiment, distance between the screen and a light source is 150 cm and the lens used has a small aperture. By moving the lens between the source and the screen, sharp images are obtained on the screen for two different positions of the lens. The focal length of the lens is 24 cm. Find the ratio of the sizes of these two images formed on the screen.

12. A neutron with kinetic energy 88 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle 90° with respect to its original direction of motion and the He^+ atom is found to be in its third excited state after collision. Find the kinetic energy (in eV) of He^+ atom after collision.

13. The circuit shown consists of two inductors of inductance L = 0.4 mH each and a capacitor of capacitance C = 2μF. The emf of battery is ε = 6V and its internal resistance r = 1Ω. Initially the switch 'S' was in position-1 for a long time. Find the maximum charge (in μC) on the capacitor after the switch 'S' is shifted to position-2.

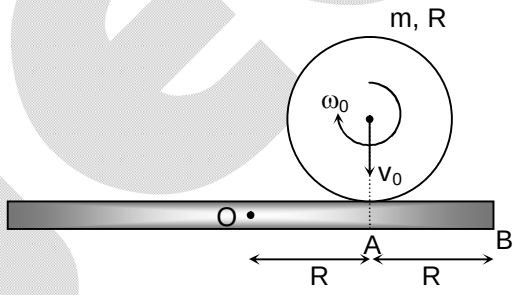


SECTION – C
(Numerical Answer Type)

*This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Paragraph for Question Nos. 14 and 15

A uniform rod of mass M = 3 kg and length ℓ = 4R = 0.80 m is hinged at its centre O and located in a vertical plane. A solid ball of mass m = 1 kg and radius R = 0.20 m having initial angular velocity ω_0 = 35 rad/s moving vertically downward with a velocity v_0 = 10 m/s collides elastically with the rod at a point A which is located at a distance R = 0.20 m from the centre 'O' of the rod as shown in the figure. The coefficient of friction between the ball and the rod is μ = 0.5. Then answer the following questions.

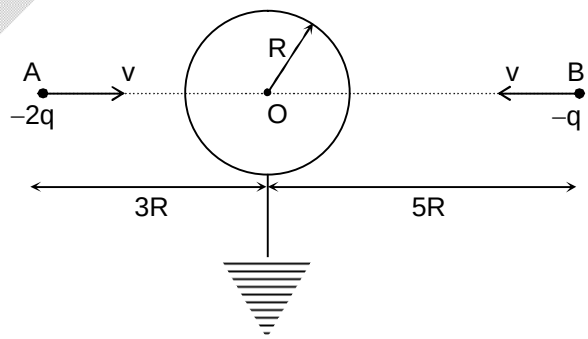


14. The angular velocity of the rod just after collisiion is ……rad/s.

15. The angualr velocity of the ball just after collision is ……..rad/s.

Paragraph for Question Nos. 16 and 17

A thin spherical conducting shell of radius R is earthed by a conducting wire. Two point charges (–2q) and (–q) initially located at points A and B start moving with the same constant speed v towards the centre 'O' of the conducting shell as shown in the figure. Then answer the following questions. (The conducting shell is having small orifices to accommodate the point charges inside the shell. Consider the conducting shell as a complete spherical shell)



16. If the current in the conducting wire at time $t = \frac{R}{v}$ is $\alpha\left(\frac{5qv}{16R}\right)$. Then the value of α is …….

17. If the current in the conducting wire at time $t = \frac{5R}{2v}$ is $\left(\frac{qv}{nR}\right)$. Then the value of n is ……..

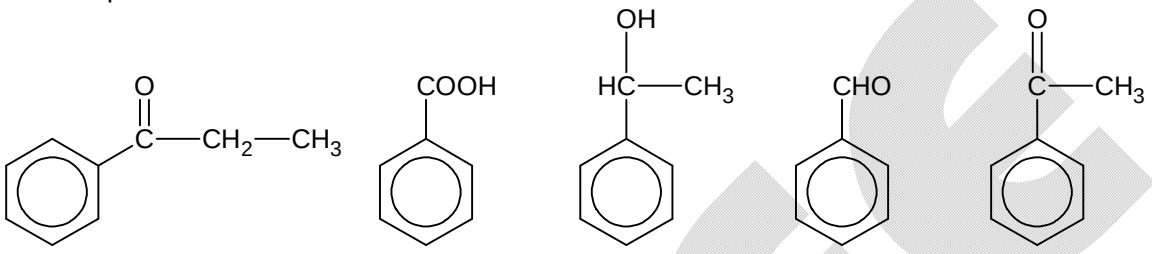
Chemistry

PART – II

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

18. The compounds shown below



(I) (II) (III) (IV) (V)

Choose the **incorrect** statement regarding these compounds:

(A) Compound I, IV and V produced red-orange precipitate when small amount of each was added to a similar volume of Brady's reagent.
(B) Only compound (III) and (V) produced a bright yellow precipitate with sodium hypoiodite solution.
(C) Compound (II) is soluble in dilute NaOH solution.
(D) Only compound I and V give optically active compounds when they react with $LiAlD_4$ and H_2O.

19. The following is a given balanced chemical reaction for the synthesis of a super conductor (A)

$$2Y_2O_3 + 8BaCO_3 + 12CuO + O_2 \xrightarrow[\text{for } 12\text{ Hr}]{850^oC} 4A + 8CO_2\uparrow$$

If neither Y nor Ba can change their common oxidation state in this reaction. What is the average oxidation state of Cu in the product.

(A) $+\frac{7}{3}$ (B) + 14

(C) $+\frac{5}{2}$ (D) $+\frac{7}{4}$

20. Solid BaF_2 is added to a solution containing 0.2 mol of $Na_2C_2O_4$ solution (2 L) until equilibrium is reached. If K_{sp} of BaF_2 and BaC_2O_4 is 10^{-8} $(mol/L)^3$ and 10^{-9} $(mol/L)^2$ respectively. Find the concentration of $[Ba^{+2}]$ in the solution. Assume addition of BaF_2 does not cause any change in volume.

(A) 2.5×10^{-5} M (B) 2.5×10^{-7} M
(C) 1×10^{-8} M (D) 4.0×10^{-6} M

21. Using the data (kcal/mol at 298 K) given below. Calculate the value of $\frac{x+y}{4}$ .....

x and y is bond energy of C – C and C – H bonds in (kcal/mol) respectively

$C(s)+O_2(g)\longrightarrow CO_2(g)$ $\Delta H=-94.0$

$H_2(g)+\frac{1}{2}O_2(g)\longrightarrow H_2O(\ell)$ $\Delta H=-68.0$

$H_2(g)\longrightarrow 2H$ $\Delta H=104$

$C(s)\longrightarrow C(g)$ $\Delta H=172$

Heat of combustion of $C_2H_6=-372.00$

Heat of combustion of $C_3H_8=-530.00$

(A) 45.25 (B) 94.60
(C) 90.50 (D) None of these

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

22. Which of the following is/are intensive properties?
(A) Specific heat (B) Heat capacity
(C) Refractive index (D) Surface tension

23. The reaction,

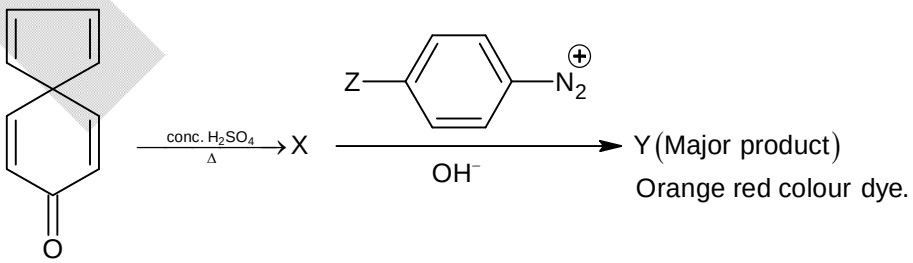
$A+B+C\longrightarrow P+Q$

was studied using method of initial rates and the following data were obtained.

C.N.	$[A]_o(M)$	$[B]_o(M)$	$[C]_o(M)$	$\frac{d[P]}{dt}(M/h^{-1})$ initial rate
1	0.01	0.01	0.01	2×10^{-3}
2	0.02	0.02	0.01	8×10^{-3}
3	0.02	0.02	0.04	1.6×10^{-2}
4	0.02	0.01	0.04	1.6×10^{-2}

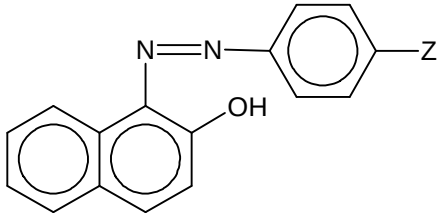
Choose the correct statement(s):
(A) Over all order of the reaction is 2.5.
(B) Over all order of the reaction is 3.0.
(C) Rate constant of the reaction is 200 $M^{-3/2}h^{-1}$.
(D) The time it will take one half of [A] to be consumed, in a reaction mixture that has initial concentration $[A]_o=0.01\,M.\ [B]_o=1.00\ M.\ [C]_o=2.00\,M$ is approximately 21 min.

24.



Choose the correct statements regarding the above reaction/products.
(A) In basic medium compound X forms 9 cannonical structure.
(B) If Z-group is electron donating rate of reaction from X to Y increases.

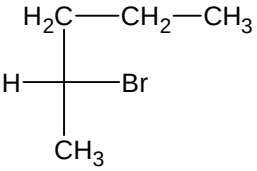
(C) If Z-group is electron withdrawing rate of reaction form X to Y decreases.

(D) Compound Y is 

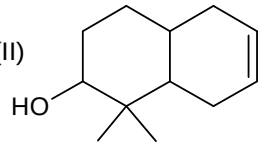
SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

25. 

(I) $\xrightarrow[\text{bromination}]{\text{Mono}}$ Number of possible chiral products is 'x'

(II) 

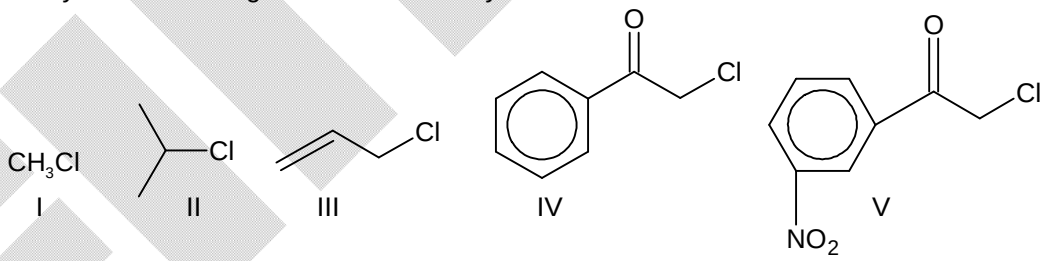
$\xrightarrow[\substack{(II)\ KMnO_4\ (aq.\ dilute)\\ 0^oC}]{(I)\ H^+/\Delta}$ Number of hydroxyl group in product is 'y' and number of (chiral) centre in end products are 'z'.

Find the value of x + y + z is……

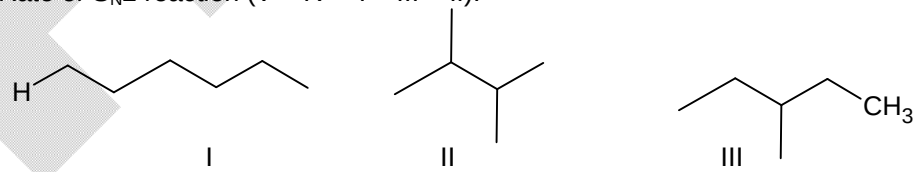
26. $A \xrightarrow[(II)\ PBr_3]{(I)\ NaBH_4} B \xrightarrow[(III)\ H_3O^+]{\substack{(I)\ Mg/ether\\ (II)\ CO_2}} C \xrightarrow{SOCl_2} D \xrightarrow[\substack{Pd/BaSO_4\\ quinoline}]{H_2}$ cis – oct – 4 – enal

Molar mass of compound A is (g/mol)……

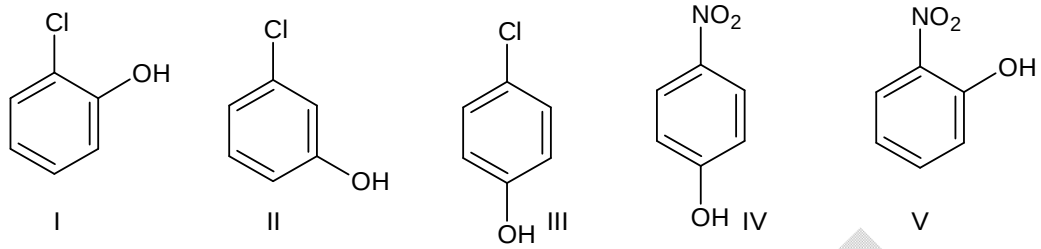
27. How many of the following orders are correctly matched:

(I) 

Rate of S_N2 reaction (V > IV > I > III > II).

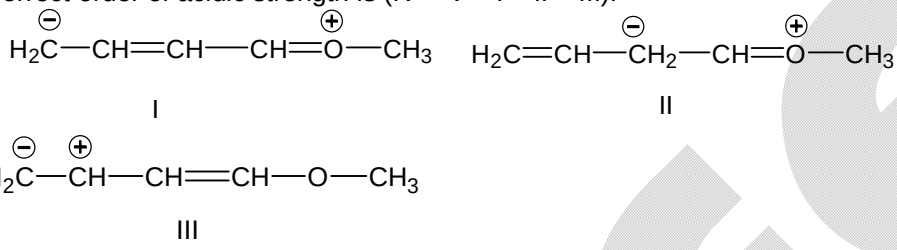
(II) 

Correct order of Boiling point (I > III > II)

(III) 

I II III IV V

Correct order of acidic strength is (IV > V > I > II > III).

(IV) 

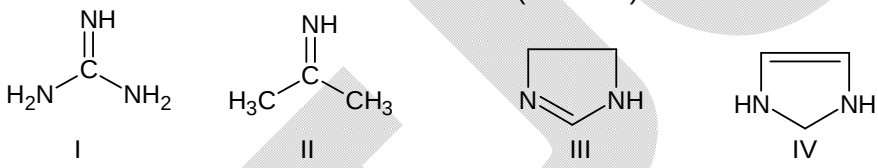
I II III

Correct order of stability is (II > I > III).

(V) C_6H_{14} C_4H_8 C_4H_6

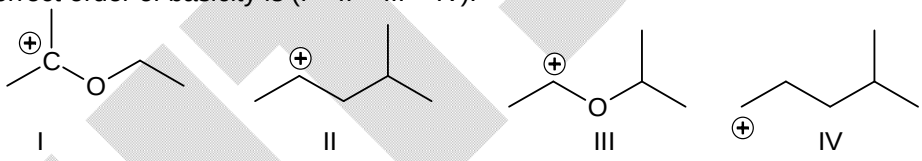
I II III

Correct order number of structural isomers is (III > II > I).

(VI) 

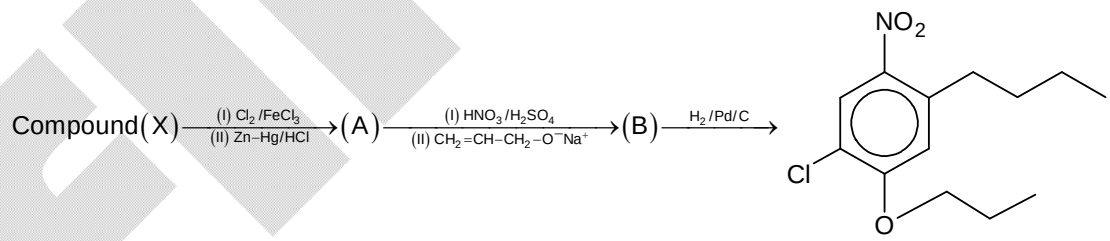
I II III IV

Correct order of basicity is (I > II > III > IV).

(VII) 

I II III IV

Correct order of stability is (I > III > II > IV).

28. 

If molar mass of compound 'X' is 'a' and degree of unsaturation is 'b' calculate the value of $\frac{2a}{b}$ ....

29. A light green colour metallic hydrated salt (X) on heating gives a white anhydrous residue Y. Y on strong heating gives a brown residue A and a mixture of gases (B and C). The gases mixture when passed through acidified permanganate discharges the pink colour and when passed through acidified $BaCl_2$ solution gave a white precipitate. Compound Y is soluble in water and its aqueous solution reacts with NO to give a dark brown compound (D). If molar mass of compound (A) is 'a' and sum of molar mass of B and C is 'b' and number of unpaired electron in (D) is 'x'.

Find the value of $\frac{a+b}{x+1}$ .....

[At. Mass of Fe = 56, Co = 58, Ni = 59, Cu = 63.5]

30. In the given statements find the value of 3x + 5 is…….
Where 'x' is the correct number of statement(s).

(I) The IUPAC name of $[Co(NH_3)_6]Cl_3$ is hexamine cobalt (III) chloride.

(II) The electron density in the xy plane in $3d_{x^2-y^2}$ orbital is zero.

(III) Silver chloride is more soluble in very concentrated NaCl solution than in pure water.

(IV) The geometries of the ammonia complexes of Ni^{+2}, Pt^{+2} and Zn^{+2} respectively are tetrahedral, square planar and octahedral.

(V) Among
$\left[Ni(CN)_4Cl_2\right]^{2-}, \left[Ni(CO)_4\right], [NiCl_4]^{2-}, [PtCl_4]^{2-}, \left[Co(NH_3)_4Cl_2\right]Cl$, $Na_3[CoF_6], Na_2O_2, CsO_2$, K_2O, O_2 and O_3.
The total number of paramagnetic compounds are 4.

(VI) In the following reaction sequence in aqueous solution the colour of Z is black.
$$S_2O_3^{2-} \xrightarrow{Ag^+} \underset{\text{Clear solution}}{X} \xrightarrow{Ag^+} \underset{\text{White}}{Y\downarrow} \xrightarrow{\text{after long time}} Z\downarrow$$

(VII) $CuCl_2$ or $(CH_3COO)_2Pb$ can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution(s).

(VIII) The addition of HCl (6 M) will precipitate $PbCl_2$ and Hg_2Cl_2 from an aqueous solution contains Hg^{+2}, Hg_2^{2+}, Pb^{+2} and Cd^{+2}.

SECTION – C
(Numerical Answer Type)

*This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Paragraph for Question Nos. 31 and 32

A modified van der Waal's equation of state, for 1.0 mole of a real gas is given as

$\left(P+\frac{\alpha}{TV^2}\right)(V-\beta)=RT$ when α and β are constant having value 4.0×10^{-3} and 2×10^{-3} respectively and value of $R = 0.0821\,L.atm/mol.K$.

31. The Boyle's temperature (in K) of gas is……..

32. If methane follows normal van der Waal's equation of state. Calculate the pressure (in atm) exerted by 1 mole of CH_4 at temperature $\frac{24}{0.821}K$ if volume occupied by CH_4 molecule is negligible (Given $a = 2\,atm\,Lit^2\,mol^{-2}$). R = 0.0821 L atm/mol K.

33. In the above reaction sequence moles of B formed is…….

34. Final mole of compound (G) is

Mathematics

PART – III

SECTION – A
(One Options Correct Type)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.*

35. If $f(x) = \tan^{-1} x^2 + \tan^{-1} x^4 + \tan^{-1} x^6 + ... + \tan^{-1} x^{2n}$ and $\int_a^{a+1} f(x)dx = \lim_{x\to\infty} \ln\left(\sum_{n=0}^{x}\int_0^x \frac{y^{n-1}\cdot e^{-\frac{y}{e}}}{e((n-1)!)^2}dy\right)$, $a > 0$, then the value of $\int_a^{a+1} f\left(\frac{1}{x}\right)dx$ is

(A) $\frac{n\pi}{2} - e^{-1}$ (B) $n\pi - e^{-1}$

(C) $\frac{n\pi}{2} - e$ (D) $n\pi - 2e$

36. A right angled triangle ABC, $B\left(\frac{2}{3}, 6\right)$ is the right angle is inscribed in a rectangular hyperbola xy = 4, the equation of common chord of the circles whose diameters are sides of the ΔABC, BC and AB, is

(A) 9x + y = 12 (B) 3x – 27y – 160 = 0

(C) 3x – y + 4 = 0 (D) 3x – 2y + 10 = 0

37. Number of real values of x satisfying $4^x + 16^x + 32^x + 144^x + \sqrt{\frac{1}{4} - x^2} = 1$

(A) 0 (B) 1

(C) 2 (D) more than 2 solutions

38. The range of 'a' for which $(a^2 - 1, a + 2)$ lies in the obtuse angle region of the lines x – 3y + 7 = 0 and x + 2y = 38 is

(A) (–8, 0) $\cup$ (4, 3) (B) (–5, 2) $\cup$ (5, 7)

(C) (–7, 0) $\cup$ (3, 5) (D) none of these

SECTION – A
(One or More than one correct type)

*This section contains **THREE (03)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

39. Which of the following is/are CORRECT?

(A) If f is a continuous function in [0, 1] for all $n \in N$

$$2n\left(1 + 2\int_0^1 \left(f(\sin x)\right)^2 dx\right) \geq \sin 2n + 8n\int_0^1 \sin(nx)\cdot f(\sin x)dx$$

(B) If $A(z_1)$, $B(z_2)$, $C(z_3)$ are vertices of a triangle such that $z_3 = \left(\frac{z_2 - iz_1}{1-i}\right)$ and $|z_1| = 3$, $|z_2| = 4$ and $|z_2 + iz_1| = |z_1| + |z_2|$, then area of triangle ABC is $\frac{25}{4}$ square units

(C) Let f, g : R $\to$ R be two bijective functions such that they are mirror image of each other w.r.t the line 4y – 17 = 0 and h(x) = f(x) + g(x), then value of $h(0) = \frac{17}{2}$

(D) If the equation $x^4 + kx^3 + x^2 + kx + 1 = 0$, $k \in R$ has exactly two distinct negative roots, then the range of k is $\left(\frac{3}{2}, \infty\right)$

40. Let $f(m) = \lim\limits_{x\to 0^+} \operatorname{cosec} x \cdot \cos^{-1}\left(\frac{m^2\operatorname{cosec}^2 x - 1}{m^2\operatorname{cosec}^2 x + 1}\right)$, $r \in N$, then

(A) $\lim\limits_{n\to\infty} \sum\limits_{r=1}^{n} f\left(2^r\right) = 2$

(B) $\left[\sum\limits_{r=1}^{1000} f\left(2 \cdot r^{\frac{2}{3}}\right)\right] = 27$; (Where [.] represents greatest integer function)

(C) $\lim\limits_{n\to\infty} \prod\limits_{r=1}^{n} \left(\frac{\left(\left(f(2r)\right)^2 + 1\right)^2}{\frac{\left(f(r)\right)^4}{4} + 1}\right) = 2$

(D) f(m) is a bijective function for $m \in N$

41. If f : R $\to$ R be a differentiable function satisfying $(f(x))^{2n+1} = x - f(x)$, $n \in N$, then

(A) f(x) is a bijective function in R

(B) $\int\limits_{-a}^{a} f^{-1}(x)\,dx$ is always zero for $a \in R$

(C) from (0, 0) two distinct tangents can be drawn to $y = f^{-1}(x)$

(D) for n = 1 the value of $\int\limits_{2}^{10} f(x) + \int\limits_{f(2)}^{f(10)} f^{-1}(x)\,dx = 18$

SECTION – B

(Numerical Answer Type)

*This section contains **SIX (06)** Numerical based questions. The answer to each question is a **NON-NEGATIVE INTEGER VALUE**.*

42. A is a 2×2 skew-symmetric matrix such that $2n\,T_r(A) + \det(A) = 8$ where $n \in N$, then the value of $\det(A^2 + \det(A)\cdot A) + 2nT_r(A) = xyz$ where xyz is a three digit natural number. Then number of ways of expressing xyz as product of two coprime positive integers is

43. Let α, β be the real roots of the equation $x^4 + 3x^3 + 2x^2 + x = 1$ and z_k be a real number $\neq \frac{1}{\alpha}, \frac{1}{\beta}$, for k = 1, 2, ..., n + 1. If $z_{n+1} - xz_{n+1}z_n - z_n = x$ when $x = \alpha$ or β, then the value of $\frac{1}{\sum\limits_{k=1}^{8} z_k} \cdot \left(\sum\limits_{r=1}^{4096} z_r\right)$ may be equal to

44. Radius of director circle of the circle passing through the points of intersection of the rectangular hyperbola xy = d and another concentric rectangular hyperbola whose axes make 45° with axis of xy = 4 may be R, then [R] (where [.] represents greatest integer function)

45. A bag contains 7 blue and white balls. Three balls are drawn at random and found to be 2 blue and one while, the probability that the bag contains 4 blue balls and 3 white balls is $\frac{p}{q}$ where p and q are coprime positive integers if xyz = pq, then the number of positive integral solution of it is

46. Given $f(x)=\frac{\sin(\pi x)-\cos(\pi x)+2}{\sqrt{x}}$ for $\frac{1}{4}\le x\le\frac{5}{4}$, the minimum of f(x) is λ, then $[\lambda]$ is equal to (Where [.] represents greatest integer function)

47. Let $C_1 : y^2 = 4x$, $C_2 : (x-4)^2 + y^2 = 8$ be curves in the Cartesian plane. Through point P on C_1 draw line l with 45-degree inclination that intersects C_2 at distinct points Q, R. Then the maximum integral value of $|PQ|\cdot|PR|$ is

SECTION – C
(Numerical Answer Type)

*This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions of numerical answer type. The answer to each question is a **NUMERICAL VALUE (XXXXX.XX).** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Paragraph for Question Nos. 48 and 49

Suppose $f(x)=a\sin x-\frac{1}{2}\cos 2x+a-\frac{3}{a}+\frac{1}{2}$, $a\in R$, $a\neq 0$

48. If $f(x)\le 0$ for any $x\in R$, then the number of possible integral values of a is

49. If $a\ge 2$ and there exists $x\in R$ such that $f(x)\le 0$, then the range of a is $[\alpha,\beta]$, where $\frac{\beta}{\alpha}$ is

Paragraph for Question Nos. 50 and 51

Let f ang g be two real-valued differentiable functions on R satisfying $\int_0^x g(t)dt=3x+\int_x^0 \cos^2 t\cdot g(t)dt$ and

$$f(x)=\lim_{\alpha\to 0}\frac{1}{\alpha^4}\int_0^{\alpha}\frac{(e^{x+t}-e^x)\ln^2(1+t)}{2t^3+3}dt$$

50. Then the value of $(f \ln(2))^{-1}$ is

51. Range of g(x) is equal to $[\alpha,\beta]$, then the value of $\alpha\beta$ is